

Self-Attracting Walk on Lattices

Jae Woo Lee¹

Department of Physics, Inha University, Inchon 402-751, Korea

Department of Physics, Clarkson University, Potsdam, NY 13699-5820, USA

Abstract

We have studied a model of self-attracting walk proposed by Sapozhnikov using Monte Carlo method. The mean square displacement $\langle R^2(t) \rangle \sim t^{2\nu}$ and the mean number of visited sites $\langle S(t) \rangle \sim t^k$ are calculated for one-, two- and three-dimensional lattice. In one dimension, the walk shows diffusive behaviour with $\nu = k = 1/2$. However, in two and three dimension, we observed a non-universal behaviour, i.e., the exponent ν varies continuously with the strength of the attracting interaction.

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¹e-mail:jwlee@munhak.inha.ac.kr

There is great interest in random walks and interacting walks[1,2,3]. The mean square displacement of a random walk, $\langle R^2(t) \rangle$ follows a power-law $\langle R^2(t) \rangle \sim t^{2\nu}$. The ordinary random walk(RW) is diffusive, with $\nu = 1/2$, in all dimensions. For a walk with repulsion, such as self-avoiding walk(SAW), the exponent ν is greater than $1/2$. Random walk on a fractal is anomalous with $\nu < 1/2$ [2,3]. Various models of interacting walks have been studied, such as true self-avoiding walk[4,5], generalized true SAW[6], the Domb-Joyce model[7], and an interacting walk with a weight factor p for each new site that the random walker visits[8,9]. A comparative study of interacting random walk models was performed by Duxbury et.al.[10,11].

Recently, Sapozhikov proposed a generalized walk in which the probability for the walker to jump to a given site is proportional to $p = \exp(-nu)$, where $n = 1$ for the sites visited by the walker at least once and $n = 0$ for other sites[12]. If $u < 0$, the walker is attracted to its own trajectory. This walk is called a self-attracting walk(SATW). Monte Carlo studies have suggested that $\nu < 1/2$ for $u = -1$ and $u = -2$ on two dimension and $1/4 < \nu < 1/3$ on three dimension[12]. However, Aarão Reis obtained non-universal behaviour of the SATW in one to four dimension using exact enumeration method[13]. Prasad et.al. concluded that the SATW in one dimension is diffusive[14].

In the present comment we report results of a Monte Carlo simulation for the self-attracting walk in one, two and three dimension. We find that SATW in one dimension is diffusive. However, SATW in two and three dimension shows non-universal behaviour. Monte Carlo simulations were performed on one dimensional lattice (10^6 -steps with 2000-configurational averages for each parameter u), square lattice (10^6 -steps with 2000-averages), and cubic lattice (10^5 -steps with 2000-averages). The lattice sizes used in this simulation were $L = 10^6$ (1D), 1024×1024 (2D) and $200 \times 200 \times 200$ (3D). We always used the periodic boundary conditions. We calculated the mean square displacement $\langle R^2(t) \rangle$ and the mean number of visited sites $\langle S(t) \rangle$.

Fig. 1 (a) shows the log-log plot of the mean square displacement against the time. We obtained the exponent by a least-square fit: $\nu = 0.500(7)$ for $u = 0$, $\nu = 0.500(9)$ for $u = -0.5$, $\nu = 0.499(8)$ for $u = -1.0$, and $\nu = 0.498(9)$ for $u = -2.0$. All the lines are parallel in the large-time limit. These results means that SATW in one dimension is diffusive and support the conclusions of Prasad et.al.[14]. This diffusive behaviour is further supported by the results of the mean number of visited sites, $\langle S(t) \rangle$, in Fig. 1 (b).

The mean number of visited sites $\langle S(t) \rangle$ shows the power law behaviour as $\langle S(t) \rangle \sim t^k$ with $k = 0.499(7)$ for $u = 0.0$, $k = 0.499(4)$ for $u = -0.5$, $k = 0.499(8)$ for $u = -1.0$, and $k = 0.498(5)$ for $u = -2.0$. At short times the slopes of the mean square displacement depend on the parameter u . However, for large times all the lines are parallel and give the same slope regardless of the parameter u . Aarão Reis concluded that the exponent ν in one dimension decreases continuously when u decreases, by the exact enumeration calculation up to $t = 30$ [13]. But their conclusion is not correct because the time steps are too short to reach the asymptotic behaviour, and the log-log plot of $\langle R^2(t) \rangle$ has curvature at short times.

The log-log plot of the mean-square displacement and the mean number of visited sites of a two dimensional SATW versus time are shown in Fig. 2 (a) and (b), respectively. The appearance of the plateau region for large times is due to the finite size of the substrate. In this region the walk touches the boundary. The exponents ν were obtained as $\nu = 0.500(3)$ for $u = 0$, $\nu = 0.472(4)$ for $u = -0.5$, $\nu = 0.404(5)$ for $u = -1.0$, and $\nu = 0.300(9)$ for $u = -2.0$. Our results are consistent with those of Aarão Reis[13], Sapozhnikov[12], and Lee[15]. Values of the exponents ν decrease continuously when u decreases. We can not observe a critical value u_c proposed Sapozhnikov so that $\nu = 1/2$ for $0 < u < u_c$ [12]. The mean number of visited sites $\langle S(t) \rangle$ shows the logarithmic correction for random walk ($u = 0$ case)[1,2]. The slopes of the log-log plots of $\langle S(t) \rangle$ decrease continuously when u decreases.

Fig. 3 (a) and (b) show a log-log plot for the mean square displacement and the mean number of visited sites for a three dimensional SATW in a cubic lattice. The plateau region is also due to the finite size of the substrate. The exponents ν were obtained as $\nu = 0.500(2)$ for $u = 0$, $\nu = 0.485(4)$ for $u = -0.5$, $\nu = 0.466(5)$ for $u = -1.0$, and $\nu = 0.294(6)$ for $u = -2.0$. The exponents ν also decrease continuously when u decreases. These results are also consistent with those of Aarão Reis[13]. But they are not consistent with the prediction of Sapozhnikov that $1/4 < \nu < 1/3$. In Sapozhnikov's argument there is an ambiguity regarding the scaling between the bulk cluster and the boundary cluster visited by the walk. We obtained the exponents k as $k = 0.996(5)$ for $u = 0$, $k = 0.993(2)$ for $u = -0.5$, $k = 0.991(5)$ for $u = -1.0$, and $k = 0.904(6)$ for $u = -2.0$. For random walk, $k = 1$ in three dimension[1,2]. We observed that the values of k decrease slowly when u decreases. These Monte Carlo results are also in good agreement with the observations of Aarão Reis[13]. The continuous decrease of the exponent

ν and k in three dimension is further confirmed by the simulation at the u -value between $u = -1.0$ and $u = -3.0$. We obtained the exponents as $\nu = 0.436(4)$, $k = 0.970(3)$ for $u = -1.5$, $\nu = 0.240(5)$, $k = 0.806(3)$ for $u = -2.5$, and $\nu = 0.190(8)$, $k = 0.776(5)$ for $u = -3.0$. These results support that there are no crossover behaviours at the range $-3.0 \leq u \leq 0$.

We calculated the exponents ν and k in one, two and three dimensions by Monte Carlo simulations. We have concluded that SATW in one dimension is diffusive. This observation supports the recent calculation of Prasad et.al.[14]. However, in two and three dimensions the exponents ν decrease continuously when u decreases. These non-universal behaviours are consistent with those of Aarão Reis[13]. Our observations are limited on regular lattices. One still has to explore the scaling behaviour on general lattices such as fractal.

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References

- [1] J.Haus and K.W. Kehr, Phys. Rep. **150**, 2631 (1987).
- [2] S. Havlin and D. ben-Avraham, Adv. Phys. **36**, 695 (1987).
- [3] A. Bunde and S. Havlin, “Fractal and Disordered Systems”, (Springer-Verlag, Berlin 1991).
- [4] D.J. Amit, G. Parisi and L. Pelit, Phys. Rev. B **27**, 1635 (1983).
- [5] A.L. Stella, S.L.A. de Queiroz, P.M. Duxbury and R.B. Stinchcome, J. Phys. A:Math. Gen. **17**, 1903 (1984).
- [6] H.C. Öttinger, J. Phys. A:Math. Gen. **18**, L363 (1983).
- [7] C. Domb and G.S. Joyce, J. Phys. C:Solid State Phys. **5**, 956 (1972).
- [8] H.E. Stanley, K. Kang, S. Redner and R.L. Blumberg, Phys. Rev. Lett. **51**, 1223 (1983).
- [9] S. Redner and K. Kang, Phys. Rev. Lett. **51**, 7129 (1983).
- [10] P.M. Duxbury, S.L.A. de Queiroz and R.B. Stinchcome, J. Phys. A:Math. Gen. **17**, 2113(1984).
- [11] P.M. Duxbury and S.L.A. de Queiroz, J. Phys. A:Math. Gen. **18**, 661 (1985).
- [12] V.B. Sapozhnikov, J. Phys. A:Math. Gen. **27**, L151(1994).
- [13] F.D.A. Aarão Reis, J. Phys. A:Math. Gen. **28**, 3851 (1995).
- [14] M.A. Prasad, D.P. Bhatia and D. Arora, J. Phys. A:Math. Gen. **28**, 3037 (1996).
- [15] J.W. Lee, J. Kor. Phys. Soc. **28**, S403 (1995).

Figure Captions

Figure 1: (a) Log-log plot of the mean square displacement versus time; (b) log-log plot of the mean number of visited sites versus time for one dimensional SATW for $u = 0$ (topmost curve), -0.5 , -1.0 , -2.0 (bottom curve).

Figure 2: (a) Log-log plot of the mean square displacement versus time; (b) log-log plot of the mean number of visited sites versus time for two dimensional SATW for $u = 0$ (topmost curve), -0.5 , -1.0 , -2.0 (bottom curve).

Figure 3: (a) Log-log plot of the mean square displacement versus time; (b) log-log plot of the mean number of visited sites versus time for three dimensional SATW for $u = 0$ (topmost curve), -0.5 , -1.0 , -2.0 (bottom curve).











